NPS-OR-97-014

NAVAL POSTGRADUATE SCHOOL
Monterey, California

Probability Models for Battle Damage Assessment
(Simple Shoot-Look-Shoot and Beyond)

by

Donald P. Gaver
Patricia A. Jacobs

August 1997

Approved for public release; distribution is unlimited.

Prepared for: Space-C2 Information Warfare, Strategic Planning Office, N6C3, Washington, DC 20350-2000

Director, J-8, The Joint Staff, Conventional Forces Analysis Div. Washington, DC 20318

Institute for Joint Warfare Analysis, NPS, Monterey, CA 93943

NAVAL POSTGRADUATE SCHOOL
MONTEREY, CA 93943-5000

Rear Admiral M. J. Evans
Superintendent

Richard Elster
Provost

This report was prepared for and funded by Space-C2-Information Warfare, Strategic Planning Office, N6C3, Washington, DC 20350-2000; Director, J-8, The Joint Staff, Conventional Forces Analysis Div., Washington, DC 20318, and the Institute for Joint Warfare Analysis, NPS, Monterey, CA 93943.

This report was prepared by:

Donald P. Gaver
DONALD P. GAVER
Distinguished Professor of
Operations Research

Patricia A. Jacobs
PATRICIA A. JACOBS
Professor of Operations Research

Reviewed by:

RICHARD E. ROSENTHAL
Chairman
Department of Operations Research

Released by:

DAVID W. NETZER
Associate Provost and Dean of Research

GERALD G. BROWN
Associate Chairman for Research
Department of Operations Research

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE
August 1997

3. REPORT TYPE AND DATES COVERED
Technical

4. TITLE AND SUBTITLE
Probability Models for Battle Damage Assessment
(Simple Shoot-Look-Shoot and Beyond)

5. FUNDING NUMBERS
N0001496WR30089

6. AUTHOR(S)
Donald P. Gaver and Patricia A. Jacobs

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Naval Postgraduate School
Monterey, CA 93943

8. PERFORMING ORGANIZATION REPORT NUMBER
NPS-OR-97-014

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Space-C2-Information Warfare, Strategic Planning Office, N6C3
2000 Navy Pentagon, Washington, DC 20350-2000

Director, J-8, The Joint Staff, Conventional Forces Analysis Div.
Rm 1D940, The Pentagon, Washington, DC 20318

Institute for Joint Warfare Analysis, NPS, Monterey, CA 93943

10. SPONSORING / MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION / AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

In spite of the precision of modern weaponry and sensor/communication systems, shots fired at targets do occasionally miss (or cause only partial damage). Consequently a sequence of more than one shot may be directed at a particular target to increase the probability of kill. The role of Battle-damage assessment (BDA) in such a setting is to make a judgment as to when further shots are actually necessary. BDA is not error-free. Probability models of the effect of battle damage assessment on shooting tactics are described and studied. Model results are presented to examine some features of tradeoffs between kill probability, p_K, and the capacity of a hypothetical BDA system to correctly judge the effect of a shot.

14. SUBJECT TERMS
Battle damage assessment (BDA); shoot-look-shoot; information war (IW); renewal reward process; probability of kill per target engaged; average number of shots per target

15. NUMBER OF PAGES
26

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	UL

PROBABILITY MODELS FOR BATTLE DAMAGE ASSESSMENT (SIMPLE SHOOT-LOOK-SHOOT AND BEYOND)

D. P. Gaver

P. A. Jacobs

0. Introduction and Summary

Battle-damage assessment (BDA) is an aspect of information war (IW) that has always promised to add to the efficiency of combat engagements. Furthermore, the capability of U.S. forces to carry out BDA in an accurate and timely manner has been and will be enhanced as increasingly sophisticated C4ISR sensor and communication systems become operational.

The purpose of this report is to introduce and develop analytical probability models for simplified BDA situations. In spite of the precision of modern weaponry and sensor/communication systems, shots fired at targets do occasionally miss (or cause only partial damage). Consequently a sequence of more than one shot may be directed at a particular target to increase the probability of kill. The role of BDA in such a setting is to make a judgment as to whether further shots are actually necessary; it has impact on both the logistics and economics of combat, and may also influence a shooter-defender's vulnerability. But BDA will not be error-free or perfect, nor will it be cost-free. Consequently this paper reports some features of tradeoffs between kill probability, p_K, and the capacity of a hypothetical BDA system to correctly judge

the effect of a shot. Sample tradeoffs are illustrated in Table 1 below, and later graphically. The reader may skip to Section 3 for a look at illustrative tradeoff graphs after examining Section 1 in which the problem addressed here is formalized.

Related work is as follows. Evans (1996), Aviv and Kress (to appear) and Manor and Kress (to appear) obtain results for models of BDA in which there is a fixed collection of targets with shoot-look-shoot tactics. Gaver, Jacobs and Youngren (1997) carry out BDA analyses in a more nearly total dynamic systems setting.

1. Simplest Formulation of a Battle-Damage Assessment Problem

We advance the following as an ultimately simple formulation.

(a) Either defensive shots are taken at a target, *or* the shots are offensive, as in a deep strike action. The probability of kill (total target destruction) on a shot is assumed to be a constant, p_K (*constancy* can be relaxed). The target is either killed on one such shot or not; there is no partial damage. This activity is generically called *Shooting*.

(b) A Battle Damage Assessment (BDA) capability is represented as follows; it is called *Looking* (or BDA):

(b-1) b_{kk} = probability that if the target is *killed* it is reported killed (no more shots are taken).

(b-2) $b_{ka} = 1 - b_{kk}$ = probability that if the target is killed it is (erroneously) reported alive. The parameters b_{kk}, and b_{ka}, are conditional probabilities, applicable only when a kill has actually occurred. Hence $b_{kk} + b_{ka} = 1$.

(b-3) b_{aa} = probability that if the target is missed, i.e. is alive after a shot, it is so reported.

(b-4) $b_{ak} = 1 - b_{aa}$ = probability that if a target is alive it is reported as killed. Again b_{aa} and b_{ak} are conditional probabilities, applicable if the target has been shot at and missed, hence is still alive.

Clearly one wants b_{kk} and b_{aa} high, i.e. each close to unity. It is possible that these probabilities are a net effect ("fusion") of several kinds of looks. It is assumed here that Shooting and Looking are independent chance events, with known and constant probabilities p_K and b_{ij}, i, j designating a, k in general. Note that this doesn't account for the sometimes reasonable possibility that $p_K(2)$, the probability of 2nd shot kill (given a first-shot miss) may exceed $p_K(1)$, the reason might be that the shooter has more time to achieve a firing solution, the target is closer, etc. On the other hand such factors might well change if the target, realizing it is under attack, takes evasive action after a first-shot miss; then $p_K(2)$ might be smaller than $p_K(1)$; subsequent shots might well differ in their probabilities of success also. It is even possible that the target-prey could turn into a predator and attack the defender, suddenly reducing p_K to zero by destroying same.

A paper that discusses similar problems, and contains further references, is Almeida, Gaver and Jacobs (1995). See also Evans (1996) and Aviv and Kress (to appear) and Manor and Kress (to appear).

Table 1 presents results of models for various shooting tactics described later in this section and in Section 2.

TABLE 1
Target Kill Probability and Mean Shots per Target as Function of Firing and Assessment Rules and Parameters

	S(1)	S(2)		SLS(2)			SLS(∞)		
p_K	$k(1)$	$k(2)$	$k(2)/2$	$s(2)$	$m(2)$	$s(2)/m(2)$	$s(\infty)$	$m(\infty)$	$s(\infty)/m(\infty)$
				$b = 0.5$					
0.3	0.3	0.51	0.26	0.41	1.50	0.27	0.46	2	0.23
0.5	0.5	0.75	0.38	0.63	1.50	0.42	0.67	2	0.34
0.7	0.7	0.91	0.46	0.81	1.50	0.54	0.82	2	0.41
0.9	0.9	0.99	0.50	0.95	1.50	0.63	0.95	2	0.48
				$b = 0.7$					
0.3	0.3	0.51	0.26	0.45	1.58	0.28	0.59	2.21	0.27
0.5	0.5	0.75	0.38	0.68	1.50	0.45	0.77	1.87	0.41
0.7	0.7	0.91	0.46	0.83	1.42	0.58	0.89	1.65	0.54
0.9	0.9	0.99	0.50	0.96	1.34	0.72	0.97	1.49	0.65
				$b = 0.9$					
0.3	0.3	0.51	0.26	0.49	1.66	0.30	0.81	2.79	0.29
0.5	0.5	0.75	0.38	0.73	1.50	0.49	0.91	1.92	0.47
0.7	0.7	0.91	0.46	0.89	1.34	0.66	0.96	1.48	0.65
0.9	0.9	0.99	0.50	0.98	1.18	0.83	0.99	1.21	0.82
				$b = 0.95$					
0.3	0.3	0.51	0.26	0.50	1.68	0.30	0.90	3.03	0.30
0.5	0.5	0.75	0.38	0.74	1.50	0.49	0.95	1.95	0.49
0.7	0.7	0.91	0.46	0.90	1.32	0.68	0.98	1.45	0.68
0.9	0.9	0.99	0.50	0.99	1.14	0.87	0.99	1.16	0.85

Legend:
S(1) means one shot is fired at each target;
S(2) means two shots are (always) fired at each target;
SLS(2) means at most two shots are fired;
SLS(∞) means that shots are fired until BDA asserts a kill;
b means probability of correct BDA ($= b_{aa} = b_{kk}$ here for simplicity only);
$s(1)$ means the long-run kill rate, 1 shot/target;
$k(2)$ means the long-run kill rate, 2 shots/target;
$s(2)$ means probability an engaged target is killed, or long-run kill rate per target; if at most 2 shots are fired per target (SLS);
$m(2)$ means mean number of shots per target (SLS);
$s(\infty)$ means probability an engaged target is killed if shots are fired until BDA asserts a kill;
$m(\infty)$ means mean number of shots per target if shoot until BDA asserts a kill.

Conclusions/Insights from Table 1

Here are some observations that can be made after viewing Table 1.

(a) For small p_K (0.3, 0.5) even poor-mediocre BDA capability (b = 0.5, 0.7) can leverage up the probability of kill per target engaged quite dramatically, and at modest price in shots per target engaged when SLS(2) is employed (shoot, look, if failure is stated, shoot once more only).

(b) While SLS(2) is less effective than is a fire-and-forget salvo of 2 shots it can be almost as good even for low p_K and b (BDA success probability), but the shot-per-target engaged economy is substantial, and this increases dramatically with both p_K and b.

(c) Use of SLS(∞), i.e. firing until the target is *reported* killed, seems unjustified for very low BDA capability (b = 0.5); there will be many wasted shots and much leakage at low p_K. This (extreme) tactic becomes much more attractive relative to SLS(2) as BDA capability increases (b = 0.7, 0.9, 0.95) particularly when p_K is relatively high (0.7 or higher). Under such conditions SLS(∞) costs only a little more than SLS(2), and much less than a salvo of 2, while leveraging up the kills per target engaged considerably when p_K is realistically moderate (0.5, 0.7).

(d) An unmodeled issue: any form of SLS may well put the firer in greater jeopardy than will salvoing 2 (or more) and evasively disappearing.

1.1 The Shoot-Look-Shoot Tactic or Decision Rule

We now describe in more detail one of the most popular and natural tactics of a system that has the option of shooting, looking, and finally moving on to another target.

Tactic: **Shoot-Look-Shoot, not more than r times (abbreviates SLS(r))**

This means that if a Look, after say, the first shot, says that kill has occurred, then no more shots are fired at that target; note that this may well be wrong, and

a valid and valuable target has escaped without further prosecution; such targets are sometimes called *leakers*, and may be active threats to protected assets. But if r shots have been fired, this is the end so far as the particular shooter is concerned. Another target is selected and the process is re-initiated. Note that we do not adjust the number of repeated shots at a target to its, perhaps gradually perceived, value, alone or in comparison with other target opportunities that may appear. Such problems will be formulated and addressed in another place. Versions of such problems have been treated in the NPS Master's Degree in Operations Research thesis by Song (1996).

1.2 Measures of Effectiveness

We mention now a number of different measures of our simple system's effectiveness.

(A) Long-run Rate of Kills per Shot (inverse of Shots per Kill). Let $K(t)$ be the random number of target kills actually achieved in t shots. Then a plausible measure of, say, an SLS(r) policy with parameters p_K and b_{ij} is the per-shot (mean) kill rate $K(t)/t$. Suppose a great many targets are engaged, so t becomes large. Then it becomes interesting to examine the *long-run kill rate* (LRKR):

$$\overline{K}(r) = \lim_{t \to \infty} \frac{E[K(t);r]}{t}.$$

It turns out that the above can be evaluated explicitly using *renewal-reward theory,* cf. Ross (1983). We quote the results subsequently.

Notice that maximizing the LRKR as a function of r or p_K, or b_{ij} may not be an optimum strategy. If r is made large in order to, say, attempt to compensate for weak BDA capability, then unnecessary "overkill" shots are likely *and* the defense system tends to waste attention that might be better spent elsewhere. This effort could be especially counterproductive if there are several/many

targets concentrated in time and space, and the objective is to prevent *any* from getting through the defense layer considered.

In a later section we present several tables and graphs that illustrate the tradeoffs between kill probability and BDA capability (probability of correct assessment of a shot's effect). These are given for selected values of Long Run Kill Rate, and refer only to the Shoot-Look-Shoot policy in which $r = 2$. However our formulas allow exploration of such tradeoffs for any values of r and kill and classification parameters.

(B) Probability of Kill per Target Engaged. This is seen to be dependent upon both the probability of kill by an individual shot, and also upon the probability with which the BDA system assesses the outcome. If, for example, b_{ak} is large then too few shots are taken at a target. See Table 1 for information as to how "good BDA" can efficiently leverage up the probability of kill per target engaged.

(C) Shots Wasted. The mean or expected number of shots or bombs wasted on already-dead targets is easily calculated under the assumptions of our BDA model.

Other measures may also be relevant and interesting.

2. Calculating the Long-Run Kill Rate (LRKR): An Application of Renewal-Reward Theory

In this section we address the evaluation of MOE(A), the LRKR in terms of the basic parameters. To do so incidentally involves evaluation of MOE(B) and other relevant quantities. The method uses the viewpoint of *renewal-reward theory*, for an exposition of which see Ross (1983), Chapter 3.

Think of each defense encounter with a new target as a *cycle* of random duration, or number of shots, $C(r)$. A new target is first fired on/engaged with

one shot and the result judged; if the verdict is that a miss has occurred another shot is fired, and its result judged, and so on until *either* (i) the judgment is that a kill has occurred or (ii) r shots have been expended, whichever occurs first. This event marks a cycle termination, whereupon another cycle is initiated (new target is prosecuted). Of course if targets occur infrequently in time the new-target encounter may be delayed, but not in terms of expended shots.

The result to be used is this. Suppose

$$K_n = \begin{cases} 1 & \text{if a target kill occurs in cycle } n; \\ 0 & \text{if no target kill occurs in cycle } n; \end{cases} \tag{2.1}$$

K_n is the reward. C_n is the duration of cycle n, measured in number of shots. It is clear that $\{C_n, n = 1, 2, \ldots\}$ is a sequence of iid random variables, as is $\{K_n, n = 1, 2, \ldots\}$, but they are not necessarily independent. Renewal-reward theory states that LRKR is given by

$$\mathrm{LRKR}(r) = \lim_{t \to \infty} \frac{E[K(t);r]}{t} = \frac{E[K_n]}{E[C_n]}. \tag{2.2}$$

Thus we need to evaluate both $s(r) = E[K_n]$ and $m(r) = E[C_n]$. A backward-equation or first-step approach can be used for both.

To evaluate $s(r) = E[K_n] = P\{K_n = 1\}$ argue that on the first shot *either* (i) the target is killed, an event of probability p_K, *or* (ii) the target is not killed and this is correctly recognized, after which the process starts over but with $r - 1$ shots to go; the probability of this latter event is $(1 - p_K)b_{aa}s(r - 1)$. Thus

$$s(r) = 1 \cdot p_K + (1 - p_K)b_{aa}s(r-1). \tag{2.3}$$

This first-order difference equation can be easily solved; subject to initial condition $s(1) = p_K$,

$$\boxed{s(r) = p_K\left[\frac{1-((1-p_K)b_{aa})^r}{1-(1-p_K)b_{aa}}\right].} \tag{2.4}$$

For $r = 2$, a usual situation,

$$s(2) = p_K[1+(1-p_K)b_{aa}] \le p_K[2-p_K]. \tag{2.5}$$

It is seen that in any case $s(r) \le 1$, and that *it doesn't depend on the kill classification probabilities b_{kk}, b_{ka},* which is initially surprising; it does depend on the probability of recognizing that an alive target remains alive after a missed shot.

Next consider $m(r) = E[C_n]$. Again condition on the first shot's outcome. If, (i), the target is killed and this is correctly recognized, or, if missed, and this incorrectly classed as a kill, then the first component of the cycle-length expectation is $1\cdot(p_K b_{kk} + (1-p_K)b_{ak})$. If, (ii), the target is missed and this correctly classified then the second component can be expressed as $[1+m(r-1)](1-p_K)b_{aa}$, since in effect the process restarts (the Markov property) but with one fewer possible shots. Finally (iii) suppose the first shot kills the target and that kill goes unrecognized; then the final expected cycle component is seen to be $[1+m^*(r-1)]p_K b_{ka}$, where $m^*(x)$ is the mean of the number of shots to either call a killed target killed, or x, whichever occurs first. Adding, we conclude that

$$\begin{aligned} m(r) &= 1(p_K b_{kk} + (1-p_K)b_{ak}) + [1+m(r-1)](1-p_K)b_{aa} + [1+m^*(r-1)]p_K b_{ka} \\ &= 1 + m(r-1)(1-p_K)b_{aa} + m^*(r-1)p_K b_{ka}, \end{aligned} \tag{2.6}$$

a first-order difference equation. The function $m^*(x)$ satisfies

$$m^*(x) = 1\cdot b_{kk} + [1+m^*(x-1)]b_{ka} = 1 + m^*(x-1)b_{ka}, \tag{2.7}$$

which is solved by simple recursion from $m^*(1) = 1$;

$$m^*(x) = \frac{1-(1-b_{kk})^x}{b_{kk}}. \tag{2.8}$$

If this is substituted into (2.6) and the latter solved subject to $m(1) = 1$ by successive substitution and series summation there results this formula:

$$\boxed{\begin{aligned} m(r) &= \frac{1-[(1-p_K)b_{aa}]^r}{1-(1-p_K)b_{aa}} \\ &+ \frac{p_K b_{ka}}{b_{kk}} \left\{ \frac{1-[(1-p_K)b_{aa}]^{r-1}}{1-(1-p_K)b_{aa}} - \frac{b_{ka}^{r-1}\left[1-((1-p_K)b_{aa}/b_{ka})^{r-1}\right]}{1-(1-p_K)b_{aa}/b_{ka}} \right\}. \end{aligned}} \tag{2.9}$$

For $r = 2$ this can be seen to equal

$$m(r) = 1 + p_K b_{ka} + (1-p_K)b_{aa} \tag{2.10}$$

An explicit, but messy formula is now available for the long-run kill rate; from (2.2) it is just

$$\mathrm{LRKR}(r) = \frac{s(r)}{m(r)}.$$

For $r = 2$ it becomes

$$\mathrm{LRKR}(2) = \frac{s(2)}{m(2)} = \frac{p_K[1+(1-p_K)b_{aa}]}{1+p_K b_{ka} + (1-p_K)b_{aa}} \tag{2.11}$$

An interesting extreme case is one in which shots are fired, or bombs dropped, until BDA asserts that a kill has occurred. This is equivalent to letting $r \to \infty$ in (2.4) to get $s(\infty)$, and also in (2.9) to get $m(\infty)$; alternatively do this latter in (2.6) and (2.8). The results:

$$s(\infty) = \frac{p_K}{1-(1-p_K)b_{aa}} \tag{2.12}$$

$$m(\infty)=\frac{1+p_K b_{ka}/b_{kk}}{1-(1-p_K)b_{aa}}. \tag{2.13}$$

These formulas are derived independently in Appendix A. From these we obtain

$$\text{LRKR}(\infty)=\frac{s(\infty)}{m(\infty)}=\frac{p_K}{1+p_K\, b_{ka}/b_{kk}}. \tag{2.14}$$

An additional measure of performance is the expected number of wasted shots per target engaged: the expected number of shots that are fired *after the target is killed*. It is

$$w(\infty)=(1-b_{kk})/b_{kk}, \tag{2.15}$$

which can be sobering if b_{kk} happens to be small.

3. Illustrations of Tradeoffs

To illustrate the message of our formulas, look at this example: for LRKR(2), i.e. Shoot-Look-Shoot, examine the tradeoff between p_K and a simplified expression of sensor-look capability: $b = b_{aa} = b_{kk}$. That is, assume that both error probabilities b_{ak} and b_{ka} are *equal* (to $1 - b$). Then fix the value of long-run kills per shot at L and examine the tradeoff between p_K and b. Figures 1, 2, and 3 depict this tradeoff for increasing L values ($L = 0.5, 0.75, 0.9$). The formula used appears at the top of each graph; it comes from (2.11) by fixing L and solving for b as a function (quadratic) of p_K.

$$b(p_K;L)=\frac{L(1+p_K)-p_K}{p_K(1-p_K)+L(2p_K-1)} \tag{3.1}$$

The lesson is that there is a tradeoff: larger b can compensate for smaller p_K, but, as required L increases, the feasible ranges for which the tradeoff exists (to realize L) decreases: *both* p_K and b must be generally higher to achieve $L = 0.9$ than they need to be to obtain $L = 0.5$. Both formulas (2.11) and (2.14) show that

kills *per shot*, L, can never exceed p_K, but kills *per target engaged*, given by $s(r)$, can become arbitrarily close to one if r is large; see (2.12) with b_{aa} approaching unity.

To further explore this last point suppose p_K is relatively low and it is desired to leverage the kills/*item targeted* (not per shot fired, or bomb dropped) to a higher level. Ways to do this are:

(a) Shoot a salvo of exactly 2 shots at the target (no BDA), or drop 2 bombs per target. Then the long-run kill rate *per target engaged* is

$$k(2) = 1 - (1 - p_K)^2. \tag{3.2}$$

The long-run cost in shots per target engaged is $f(2) = 2$; in general if a fixed number of shots, r, is fired $f(r) = r$.

(b) Shoot-Look-Shoot (2). The long-run kill rate per target engaged is

$$s(2) = p_K\left[1 + (1 - p_K)b\right] \tag{3.3}$$

and the mean number of shots per target engaged is

$$m(2) = 1 + p_K(1 - b) + (1 - p_K)b. \tag{3.4}$$

Since there is always the probability $1 - b$ of making an error and not shooting a needed second shot (or shooting a superfluous one), the probability of target kill $s(2)$ is never greater than $k(2)$, being equal to it only when $b_{aa} = 1$. (There is no BDA, so there are no BDA errors.)

(c) Shoot-Look-Shoot (∞). Here we get for the probability that a target is killed from expression (2.12),

$$s(\infty) = \frac{p_K}{1 - (1 - p_K)b} \tag{3.5}$$

which can become arbitrarily close to unity for fixed p_K if $b \to 1$. The mean number of shots per target engaged is, from (2.13),

$$m(\infty) = \frac{1 + p_K((1-b)/b)}{1 - (1-p_K)b} \tag{3.6}$$
$$= \frac{b + p_K(1-b)}{b(1-(1-p_K)b)}.$$

If the target or threshold desired kill probability is $\bar{k} < 1 - (1-p_K)^2$ then option (a) will achieve it, but at a price of unnecessary shots. E.G. if $\bar{k} = 0.85$ then a lower threshold value for p_K is $p_K = 0.61$ but at a cost of 2 shots per target. If option (b) is adopted with $b = 1.0$ and if $p_K = 0.61$ then $\bar{k} = 0.85$ is achieved but at the cost in shots per target of $m(2) = 2 - p_K = 2 - 0.61 = 1.39$, decisively below 2. If $p_K = 0.7$ then a BDA success rate of at least $b = 0.71$ is required, and the cost in shots per target is $m(2) = 1 + 0.7(1 - 0.71) + 0.3(0.71) = 1.42$, still well below 2.

Discussion

The leverage of per-target kill probability by good BDA (relatively high $b = b_{aa} = b_{kk}$) is well-illustrated in Table 1, e.g. by observing the effect of increasing b on small p_K, $p_K = 0.3$, for SLS(2) and SLS(∞): even for $b = 0.7$ the probability of kill per target engaged is nearly doubled and in less than an average of 2 shots (SLS(2); 1.6), or a little more (SLS(∞); 2.2). Even a relatively small b-value, e.g. $b = 0.5$, has a noticeable effect, doing almost as well using SLS(2) and S(2) and at 75% of the number of shots.

In many ways this analysis is oversimplified, and is clearly incomplete. For instance, we do not here consider the delay and traffic handling capacity of the BDA service system, nor its cost. In Appendix B we do specify some convenient, if tentative, analytical expressions for probability levels as a function of cost of system acquisition. Further such issues will be addressed in subsequent work.

Appendix A

Self-Contained Derivation of Mean Cycle Length When $r = \infty$ (Shoot Until Target Judged Killed)

Let $m_a(\infty)$ be the mean number of shots fired (bombs dropped) until an initially active (or alive) target is *judged dead* (the judgment may be in error). Let $m_k(\infty)$ be the mean number of shots fired until a killed target is (finally) *judged dead*. Let $s_a(\infty)$ be the probability that an initially active (new) target is dead when it is *judged dead*, and let $\bar{s}_a(\infty)$ be the probability that an initially active target is *not dead* when it is *judged dead*; this is the probability that leakage occurs.

Start with the mean cycle length for new targets, $m_a(\infty)$. Then, conditionally on the outcome of the first shot and its judged effect,

$$m_a(\infty) = \begin{cases} 1, & \text{with prob. } p_K b_{kk} + (1-p_K)b_{ak} \\ 1 + m_a(\infty), & \text{with prob. } (1-p_K)b_{aa} \\ 1 + m_k(\infty), & \text{with prob. } p_K b_{ka} \end{cases} \tag{A.1}$$

$$\therefore m_a(\infty) = 1 + m_a(\infty)(1-p_K)b_{aa} + m_k(\infty)p_K b_{ka} \tag{A.2}$$

Similarly,

$$m_k(\infty) = \begin{cases} 1, & \text{with prob. } b_{kk} \\ 1 + m_k(\infty), & \text{with prob. } b_{ka} = 1 - b_{kk} \end{cases} \tag{A.3}$$

$$\begin{aligned} \therefore m_k(\infty) &= 1 + m_k(\infty)(1-b_{kk}) \\ &= 1/b_{kk} \end{aligned} \tag{A.4}$$

$$\begin{aligned} &\therefore m_a(\infty)\left[1-(1-p_K)b_{aa}\right] = 1 + p_K b_{aa} m_k(\infty) \\ &\therefore m_a(\infty) = \frac{1 + p_K b_{aa} \cdot {}^{1}\!/_{b_{kk}}}{1-(1-p_K)b_{aa}}. \end{aligned} \tag{A.5}$$

Similar equations can be written for $s_a(\infty)$ and $\bar{s}_a(\infty)$:

$$s_a(\infty) = \begin{cases} 1 & \text{with prob. } p_K \\ s_a(\infty) & \text{with prob. } (1-p_K)b_{aa}. \end{cases}$$

Consequently,

$$\begin{aligned} s_a(\infty) &= 1 \cdot p_K + s_a(\infty)\left[(1-p_K)b_{aa}\right] \\ &= \frac{p_K}{1-\left[(1-p_K)b_{aa}\right]} \end{aligned}$$

$$\bar{s}_a(\infty) = \begin{cases} 1 & \text{with prob. } (1-p_K)b_{ak} \\ \bar{s}_a(\infty) & \text{with prob. } (1-p_K)b_{aa}. \end{cases}$$

Consequently,

$$\begin{aligned} \bar{s}_a(\infty) &= 1 \cdot (1-p_K)b_{ak} + \bar{s}_a(\infty) \cdot (1-p_K)b_{aa} \\ &= \frac{(1-p_K)b_{ak}}{1-(1-p_K)b_{aa}}. \end{aligned}$$

These agree with results obtained by letting $r \to \infty$ in our previous formulas.

Appendix B

Parametric Models for the Cost of Achieving Probabilities

Suppose a higher value of kill probability, p_K, or of correct BDA, b_{kk} and/or b_{aa}, can be obtained at increased acquisition cost, D (in dollars). It is convenient to represent the cost-related payoff in terms of probabilities by parametric cost functions. Here are some possibly useful examples; they can be recognized as *logistic* transformations:

$$p_K(D) = \frac{p(\infty)\left(D/D_0(K)\right)^a}{1+\left(D/D_0(K)\right)^a}; \tag{B.1}$$

$a > 0$, $D_0(K) > 0$, $0 \leq p(\infty) \leq 1$. Notice that if acquisition cost becomes large ($D \to \infty$) then kill probability reaches a limit, $p(\infty)$, which is no greater than 1; if $D = D_0$ the resulting p_K-value is 1/2 of its ultimate. The parameter a controls the sharpness of the response of $p_K(D)$ to increases in expenditure, D: if a is very small (e.g. 1/2) the approach to $p(\infty)$ is quite slow; if a becomes large, expenditures below D_0 have small effect, while if above D_0 they produce considerable payoff.

Similar models can be hypothesized for the C4ISR assets that generate BDA:

$$b_{kk}(D) = \frac{b_{kk}(\infty)\left(D/D_0(kk)\right)^b}{1+\left(D/D_0(kk)\right)^b} \tag{B.2}$$

and

$$b_{aa}(D) = \frac{b_{aa}(\infty)\left(D/D_0(a)\right)^c}{1+\left(D/D_0(a)\right)^c}. \tag{B.3}$$

The constants play the same roles as those for (B.1). There is no need that any be the same.

It is a straightforward non-linear optimization problem to allocate expenditures to elements of a defense system that will, for example, maximize the long-run expected number of targets killed *per dollar expended*.

References

Almeida, R., Gaver, D.P., and Jacobs, P.A., "Simple probability models for assessing the value of information in defense against missile attack," *Naval Research Logistics*, **42** (1995), pp. 535-547.

Aviv, Y. and Kress, M., "Evaluating the effectiveness of shoot-look-shoot tactics in the presence of incomplete damage information." To appear in *Military Operations Research.*

Evans, D.K., "Bomb damage assessment and sortie requirements," *Military Operations Research*, **2** (1996), pp. 31-35.

Gaver, D.P., Jacobs, P.A., and Youngren, M.A., "Analytical models for battle-space information war (BAT-IW)." To appear.

Manor, G. and Kress, M., "Optimality of greedy shooting strategy in the presence of incomplete damage information." To appear in *Naval Research Logistics.*

Ross, S.M. (1983) *Stochastic Processes.* John Wiley, New York.

Song, S-J (1996) "Probability models for assessing the value of battle damage assessment in the defense against sequential theater missile attacks," Naval Postgraduate School, M.S. Thesis.

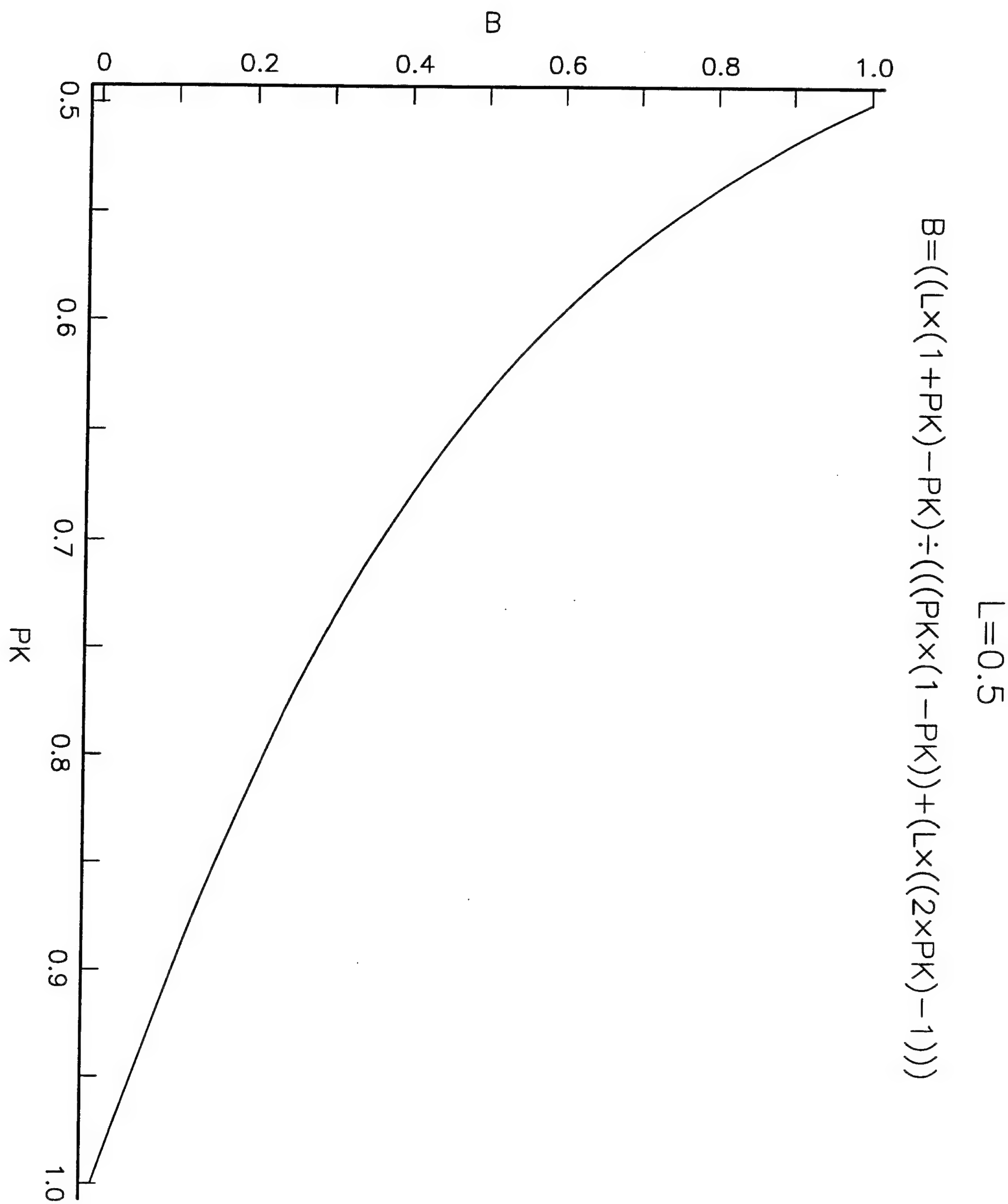
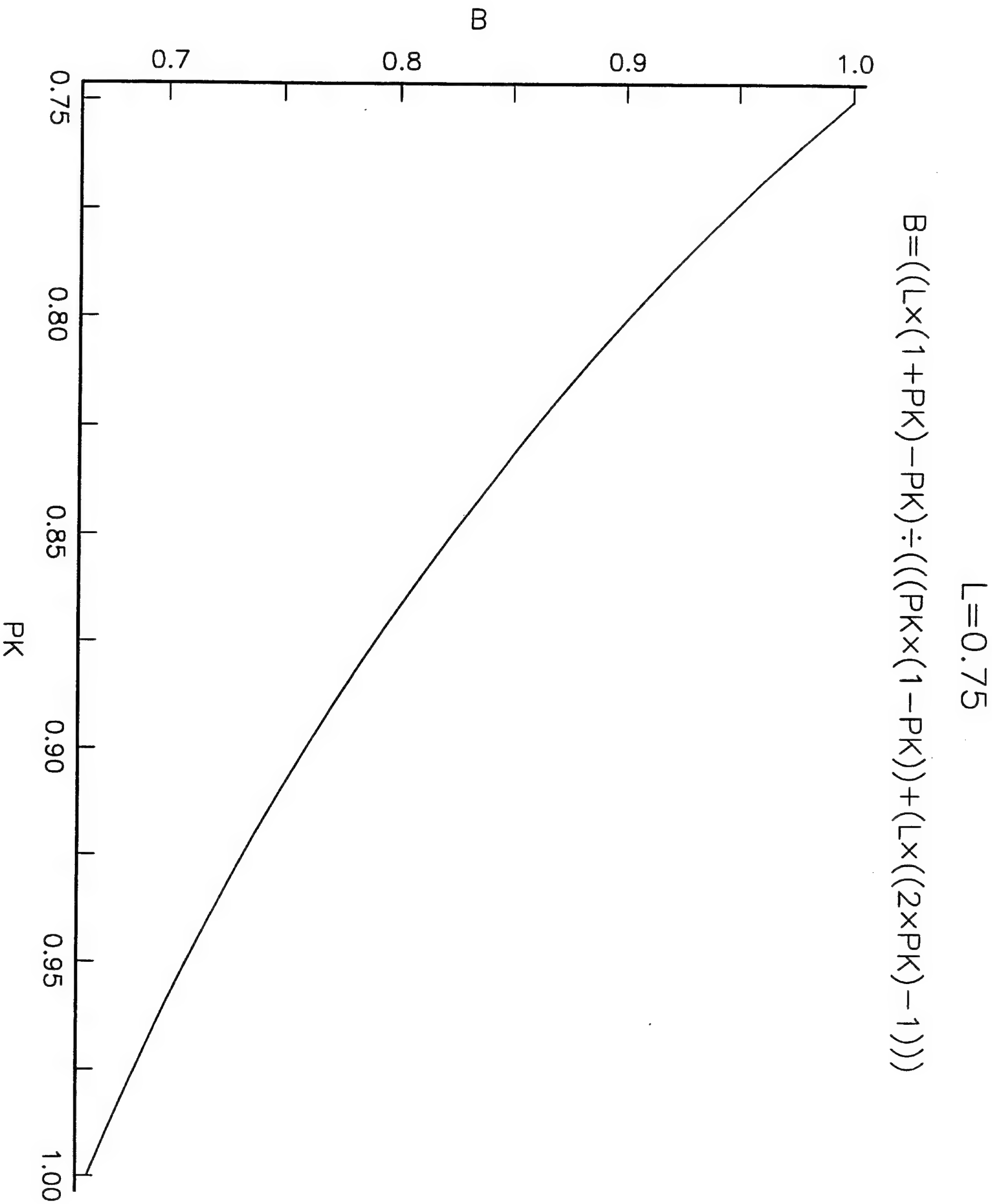


Figure 1



L=0.75
B=((L×(1+PK)−PK)÷((PK×(1−PK))+(L×((2×PK)−1)))
B
0.7
0.8
0.9
1.0
PK
0.75
0.80
0.85
0.90
0.95
1.00

Figure 2

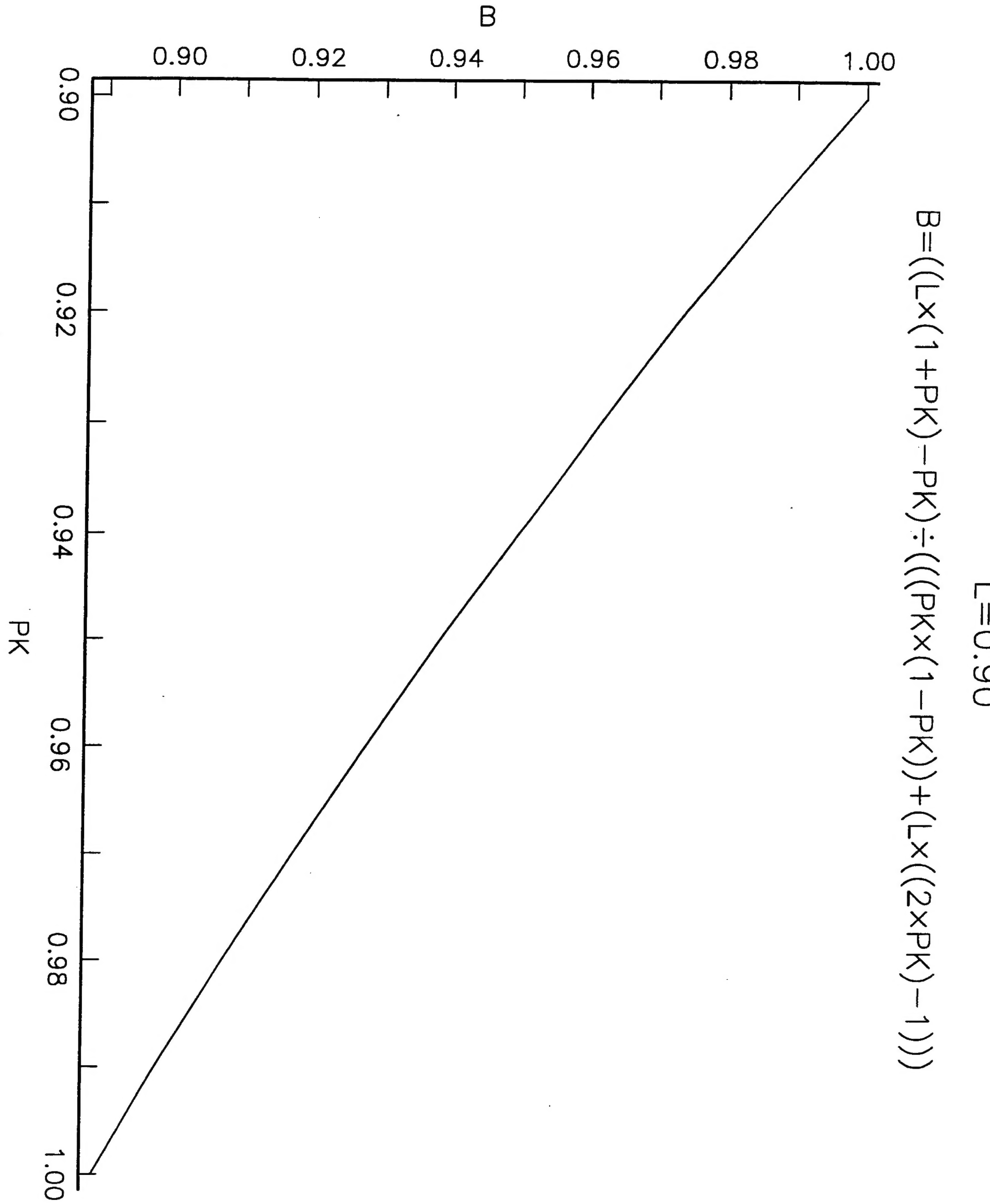

L=0.90
B=((L×(1+PK)−PK)÷((PK×(1−PK))+(L×((2×PK)−1)))
B
0.90
0.92
0.94
0.96
0.98
1.00
PK
0.90
0.92
0.94
0.96
0.98
1.00

Figure 3

INITIAL DISTRIBUTION LIST

1. Research Office (Code 09) 1
   Naval Postgraduate School
   Monterey, CA 93943-5000

2. Dudley Knox Library (Code 013) 2
   Naval Postgraduate School
   Monterey, CA 93943-5002

3. Defense Technical Information Center 2
   8725 John J. Kingman Rd., STE 0944
   Ft. Belvoir, VA 22060-6218

4. Prof. Donald P. Gaver (Code OR/Gv) 5
   Naval Postgraduate School
   Monterey, CA 93943-5000

5. Prof. Patricia A. Jacobs (Code OR/Jc) 5
   Naval Postgraduate School
   Monterey, CA 93943-5000

6. Prof. Dan Boger 1
   C3I Academic Group
   Naval Postgraduate School
   Monterey, CA 93943-5000

7. Dr. J. Abrahams 1
   Code 111, Room 607
   Mathematical Sciences Division, Office of Naval Research
   800 North Quincy Street
   Arlington, VA 22217-5000

8. Prof. D. R. Barr 1
   Dept. of Systems Engineering
   U.S. Military Academy
   West Point, NY 10996

9. Center for Naval Analyses 1
   4401 Ford Avenue
   Alexandria, VA 22302-0268

10. Mr. Mike Davis 1
R55
9800 Savage Road
Ft. Meade, MD 20755

11. Dr. Paul K. Davis 1
RAND Corporation
1700 Main St.
Santa Monica, CA 90406

12. Dr. Bruce W. Fowler 1
Technical Director, US Army
Attn: AMS MI-RD-AC
Redstone Arsenal, AL 35898-5242

13. Dr. Arthur Fries 1
Institute for Defense Analysis
1800 North Beauregard
Alexandria, VA 22311

14. Dr. Neil Gerr 1
Office of Naval Research
Arlington, VA 22217

15. Mr. Koh Peng Kong 1
OA Branch, DSO
Ministry of Defense
Blk 29 Middlesex Road
SINGAPORE 1024

16. Dr. Moshe Kress 1
CEMA
P.O.B. 2250 (TI)
Haifa 31021 ISRAEL

17. Dr. James R. Maar 1
National Security Agency
9608 Basket Ring
Columbia, MD 21045-0689

18. COL R.S. Miller 1
Institute for Defense Analysis
1800 North Beauregard
Alexandria, VA 22311

19. Mr. Vincent D. Roske, Jr. 1
The Joint Staff, J8
The Pentagon
Washington, DC 20318-8000

20. CDR Kevin Schaaff 1
SEW Strategic Planning Office, N6C5
2000 Navy Pentagon, Rm 5C633
Washington, DC 20350-2000

21. Wagner & Associates 1
Station Square One
Paoli, PA 19301

22. Dr. Howard L. Wiener 1
SEW Strategic Planning Office, N6C5
2000 Navy Pentagon, Rm 5C633
Washington, DC 20350-2000